**Home Work 3**

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1. Let *S* be defined by

*S* = .

Prove that *S* is a subspace of **R**2.

2. Let *S* be defined by

*S* = .

Determine whether *S* is a subspace of **R**2.

3. Let *u* and *v* be vectors in **R**2. Show that span{*u*, *v*}= **R**2 if and only if neither is a scalar multiple of the other. Interpret this result geometrically.

4. If *u* , *v* = and *w* = , then determine whether *w* ∈ span{*u*, *v*}.

5. In each of the following determine whether the first matrix is a linear combination of the other matrices:

(a) , , ,

(b) , , ,

(c) , , ,

6. Let *u* and *v* span a subspace *U* of a vector space *V*. Let *a* and *b* be nonzero scalars. Show that *au* and *bv* also span *U*.

7. Let *u*. *v*, *w* span a vector space *V*. Show that the vectors *u*. *v*, *u* + *v* + *w* also span *V*.

8. Let *u*. *v*, *w* span a vector space *V*. Show that the vectors *u*. *u* + *v*, *u* + *v* + *w* also span *V*.

9. Show that the vectors

, ,

are linearly dependent in **R**2.

10. Determine whether the following sets of vectors are linearly dependent or independent.

(a) {(–1,2),(2, –4)}

(b) {(–1, 3 ), ( 2, 5)}

(c) {(1, –2, 3), (–2, 4, 1), (–4, 8, 9)}

(d) {(1, 0, 2), (2, 6, 4), (1, 12, 2)}

(e) {(1, 2, 5), (1, –2, 1), (2, 1, 4)}

(f) {(1, 1, 1), (–4, 3, 2), (4, 1, 2)}

11. Show that the following sets of vectors are linearly dependent in **R**3.

(a) {(2, –1, 3), (–4, 2, –6), (8, 0, 1)}

(b) {(1, –2, 3), (7, 4, –2), (3, –6, 9)}

(c) {(5, 2, –3), (3, 0, 4), (–3, 0, –4)}

(d) {(1, 1, 1), (2, 2, 2), (0, 1, 5)}

12. Find values of *t* for which the following sets are linearly dependent.

(a) {(–1, 2), (*t*, –4)}

(b) {(3, *t*), (6, *t* – 1)}

(c) {(2, –*t*), (2*t* + 6, 4*t*)}

13. Prove that the subspace of **R**3 generated by the vectors (–1, 2, 1), (2, –1, 0), (1, 4, 3) is a two-dimensional subspace of **R**3. Give a basis for this subspace.

14. Prove that the vector (1, 2, -1) lies in the two-dimensional subspace of **R**3 generated by the vectors (1, 3, 1) and (1, 4, 3).

15. Prove that the vector (2, 1, 4) lies in the two-dimensional subspace of **R**3 generated by the vectors (1, 0, 2) and (1, 1, 2).

16. Prove that the vector (-3, 3, -6) lies in the one dimensional subspace of **R**3 generated by the vector (2, -2, 4).

17. Does the vector (1, 2, –1) lie in the subspace of **R**3 generated by the vectors (1, -1, 0) and (3, -1, 2)?

18. Find a basis for **R**2 that includes the vector (1, 2).

19. Find a basis for **R**3 that includes the vectors (1, 1, 1) and (1, 0, –2).

20. Find a basis for **R**3 that includes the vectors (-1, 0, 2) and (0, 1, 1).

21. Show that the following sets of vectors form bases for **R**3, and then express the standard basis vectors: *e*1, *e*2, *e*3 in terms of these:

a) *u*1 = , *u*2 = , *u*3 = ;

b) *v*1 = , *v*2 = , *v*3 = .